#13

ABL **BC Calculus Quiz #12 CA**

More Integration

Dr. Wisniewski Spring 2020

10pts

Name Jaiboobü ti **Squrtle**

Instructions: Solve each of the problems below. Do your work on another sheet of paper but please show your work (for partial credit) and box or circle your answers. **A calculator is permitted on this portion of the quiz.**

1. A medium-sized, stainless-steel tank initially contains 100 gallons of grape juice. Additional grape juice is then pumped into the tank for 20 minutes at a rate of $R(t) = 20t^2e^{-t/2}$ gallons per minute for $0 \le t \le 20$ min. Starting at the same time, juice is pulled from the bottom of the tank and sent through a filtration unit at a constant rate of 12 gallons/min. This removal/filtration continues until the tank is empty.
   a. How much juice is pumped into the tank in first 10 minutes?
   b. What volume of juice (in gallons) is in the tank at $t = 20$ minutes?
   c. From when the pump was first turned on ($t = 0$), how long will it take for the tank to completely empty?
   d. At what time t is the volume of juice in the tank a maximum? The tank has a maximum capacity of 300 gallons. Is this a large enough tank to support this operation? Justify!

a. Let $Q(t)$ = the vol. of juice in gallons that has entered the tank thru time t. $Q(0)=0$ $Q'(t)=R(t)$ (2pts)

$\therefore Q(10)-Q(0)=\int_0^{10} R(t)\,dt = 20\int_0^{10} t^2e^{-t/2}\,dt \overset{\text{FnInt}}{=}$ **200.111 gal** +1

b. Let $V(t)$ be the net volume of juice in the tank in gals at time t. $V'(t)$ = rate in − rate out +2 $V(0)=100$ (3pts)

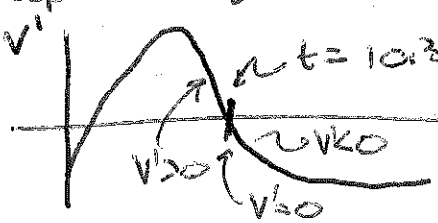
$V'(t)=20t^2e^{-t/2}-12$

$V(20)-V(0)=\int_0^{20}(20t^2e^{-t/2}-12)\,dt$; $V(20)\overset{\text{FnInt}}{=}$ **179.114 gal** +1

c. time 'til empty $= 20 + \frac{179.114\text{ gal}}{12\text{ gal/min}} =$ **34.926 min** (1pts) −1/2 if wrong

d. $V(t)$ is a max when $V'(t)=0$ and V' goes from + to neg. on graph

graph $V'(t)=20t^2e^{-t/2}-12$ vs. t for $0<t<35$ (4pts)



max volume occurs @ $t=10.382$ min +1 justifying +1

$V(10{,}382)=100+\int_0^{10.382}(20t^2e^{-t/2}-12)\,dt$ +1

$V(10{,}382)\overset{\text{FnInt}}{=}$ 260.389 gal SO TANK IS LARGE ENUF! +1